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The Distributed-mode loudspeaker (DML) as a broad-band acoustic radiator

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Abstract

The principles of a new class of acoustic radiator (DML) are described and the counter-intuitive result for broad-band, frequency independent acoustic radiation established. It is demonstrated that a low-loss panel with optimal modal distribution produces a flat power response. A simple mechanical model is presented to calculate the mean velocity within the panel as a function of frequency and intrinsic properties.

0 Introduction

The use of bending waves as a source of acoustic radiation requires a multi-discipline approach. An understanding of both structural mechanics and acoustics is necessary, and many of the results are counter-intuitive. Much of the under-lying theory is not new, indeed bending waves in panels have been the subject of study for at least two centuries. Recent advances in materials technology, and the availability of Finite Element Analysis (FEA) packages with acoustics, have enabled new ideas to be tried and tested. The Distributed Mode Loudspeaker (DML) is just such a new idea.

In the disciplines of acoustics and electronics, we are used to dealing with second order, partial differential equations - or at least their implications. Laplace's equation is used for static fields, and the wave equation for dynamic fields. In particular, the wave equation (or its steady-state variant, the Helmholz equation), with its lack of dispersion and constant velocity, describes vibrating strings and stretched diaphragms, and their associated acoustic radiation.

In contrast, bending waves require a fourth order, partial differential equation with complicated boundary conditions to describe their behaviour. To quote Lord Rayleigh [1] "for a free plate the latter part of the problem is indeed of considerable difficulty". The exact mathematical forms "defied analysis for many years". The correct fourth order differential equation was first presented in 1815 by Sophie Germain, but her choice of boundary conditions had to be corrected by Kirchhoff in 1850. The problem requires even higher orders of differential equation if the plate is not flat. These high order equations are more familiar to structural engineers, who are used to dealing with bending beams and plates. Many results are well known, and documented in reference works such as Roark [2] or Blevins [3]. It is, perhaps, because of the complicated nature of the mathematics, that bending wave transducers have only recently become a commercial reality [4].

New understanding of the relationships between the mechanical and acoustical properties of vibrating panels has resulted in the birth of a new class of acoustic radiator - the distributed-mode loudspeaker. Optimisation techniques exist, whereby stiff, light panels can be designed to have very uniform modal density, the pre-requisite for distributed-mode behaviour. This paper begins with a résumé of the basics of the ubiquitous Rice-Kellogg mass controlled, pistonic loudspeaker, with which many will be familiar. It then describes in an analogous way, the basics of a distributed-mode loudspeaker.

1 Glossary of symbols

Panel parameters

E = Young's modulus, Pa
ρ = mass density, kg/m^3
ν = Poisson's ratio
h = thickness of panel, m
B = bending rigidity of panel, Nm
μ = mass per unit area of panel, kg/m^2
Z_p = mechanical impedance of panel, kg/s

Note that for an isotropic panel $B = \frac{1}{12}\frac{E}{(1-\nu^2)}h^3$ and $\mu = h\rho$

Acoustic parameters

P = acoustic power, W
U = volume velocity, m^3/s
R_r = acoustic radiation resistance, kg/s
X_r = acoustic radiation reactance, kg/s
Z_r = acoustic radiation impedance, kg/s
R1 = acoustic radiation resistance at frequency of 1 rad/s, kg/s
ω = angular frequency, rad/s
ρ_0 = density of air, kg/m^3
c = speed of sound in air, m/s
k = acoustic wave number = ω/c, m^{-1}
a = radius of circular piston

Mechanical parameters

x, x_p, x_m = displacements, m
u, u_p, u_m = velocities, m/s
F = force, N

General

bar over variable = rms value, e.g. $\bar{u}$
single underline = vector, e.g. $\underline{x}$
double underline = matrix, e.g. $\underline{\underline{M}}$
$O(x^n)$ = terms involving powers of x greater than or equal to n

2 Overview of the traditional piston-based loudspeaker

A piston-based loudspeaker is a "mass-controlled" device, implying that a constant force will produce a constant acceleration. The requirement for constant acceleration is a direct result of the frequency dependent real part of the radiation impedance of the piston [5].

$$Z_r = R_r + jX_r \tag{1}$$

$$R_r = \pi a^2 \rho_0 c R_1(2ka) \tag{2}$$

$$X_r = \pi a^2 \rho_0 c X_1(2ka) \tag{3}$$

where $R_1(x) = 1 - 2J_1(x)/x$ which is $O(x^2)$ for small x, and $X_1(x)$ is O(x) for small x

Force, velocity and power are related by

$$F = Z_r U, \quad P = \frac{1}{2} U^2 R_r \tag{4}$$

In order to achieve a frequency independent transfer of power, velocity must be inversely proportional to frequency, or acceleration must be constant. The "flat power" response of the piston is limited to ka << 1. For ka >> 1, the radiation impedance is approximately constant and equal to $\pi a^2 \rho_0 c$, resulting in a second order roll-off in the power response of the loudspeaker. It is a convenient coincidence that the on-axis pressure response is also frequency independent, even when this directivity is taken into account.

3 Equivalent circuit of the traditional piston-based loudspeaker

To illustrate the constant power result, a simplified representation using the impedance analogue [6] is given in Figure 1. The equation of motion for this system is

$$\left(M + M_r\right)\frac{du}{dt} + R_r u = F \quad \text{or} \quad \left(j\omega\left(M + M_r\right) + \omega^2 R1\right)u = F \tag{5}$$

The delivered power is given by

$$P = \left|\overline{u}\right|^2 R_r = \frac{1}{2}|u|^2 \omega^2 R1 \tag{6}$$

Combining (5) and (6), we get

$$P = \omega^2 \frac{F^2}{2} \frac{R1}{\left|j\omega\left(M + M_r\right) + \omega^2 R1\right|^2}$$

$$P = \frac{F^2}{2} \frac{\omega^2 R1}{\omega^2\left(M + M_r\right)^2 + \omega^4 R1^2} \tag{7}$$

Using equation (2), setting $\omega = 1$, and expanding for a, we get the following expression for R1.

$$R1 = \pi a^2 \rho_0 c\left(1 - \frac{c}{a} J_1\left(\frac{2a}{c}\right)\right) \approx \frac{\pi \rho_0 a^4}{2c} - O\left(a^6\right) \tag{8}$$

So R1 is very small. Also $M_r << M$, hence we can write for frequencies where ka << 1

$$P \approx \left(\frac{F}{M}\right)^2 \frac{R1}{2} \approx \left(\frac{F}{M}\right)^2 \frac{\pi \rho_0 a^4}{4c} \tag{9}$$

For high frequencies or large a, R_r is constant at $\pi a^2 \rho_0 c$, so the power is no longer frequency independent, and is given by

$$P = \frac{F^2}{2}\frac{R_r}{\omega^2(M+M_r)^2+R_r^2} \approx \frac{F^2}{2R_r}\frac{R_r^2/M_m^2}{\omega^2+R_r^2/M_m^2} \tag{10}$$

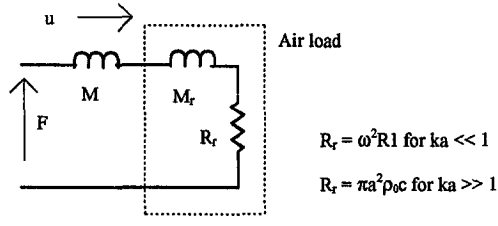


Figure 1. Impedance analogue model of a mass controlled radiator.

4 Overview of a Distributed Mode Loudspeaker (DML)

A DML is identified by the fact that its radiation is due to uniformly distributed free vibration in a stiff, light panel, and not to pistonic motion. Because bending waves are dispersive (the wave velocity is a function of frequency) [7], a good approximation is to consider the panel as a randomly vibrating area. The radiation intensity from such an area is shown in Morse & Ingard [8] to depend on the square of the mean velocity, and hence the requirement is for constant velocity. In order to achieve this constant velocity with a constant force, the mechanical impedance must be resistive. A panel operating in bending waves meets this criterion [9]. Expressions for bending wave velocity and mechanical impedance are quoted below.

$$u(\omega) = \sqrt{\omega\sqrt{B/\mu}} \qquad Zm = 8\sqrt{B\mu} \tag{11}$$

In order to develop a mathematical model of any physical system, it is necessary to make some assumptions. Because we are considering the DML to be randomly vibrating, the existing motion of the panel will be uncorrelated to any new input being applied, and therefore looks like an infinite plate. Additionally, because the panel has low mechanical loss, we can suppose that all the energy supplied to the panel will eventually be dissipated by acoustic radiation. This set of assumptions has been shown to give useful results, and is correct in as much as the radiated pressure is proportional to the velocity in the panel. Thus, to calculate the radiated acoustic power, we need only to calculate the mechanical power in the panel.

5 Equivalent circuit of a Distributed Mode Loudspeaker (DML)

Given that the DML is a resistance controlled device, and that we do not need to consider the acoustic radiation in detail, we can develop an equivalent circuit from Figure 2. This represents a simplified version of the "inertial magnet driver" application used by New Transducers Limited in their white paper [10].

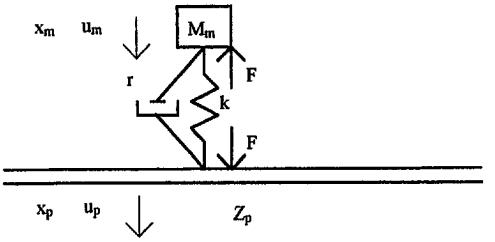


Figure 2. Mechanical elements and forces for a panel driven by a damped mass-spring oscillator.

The coupled equations of motion are given in (12) and (13) below.

$$M_m \frac{d^2 x_m}{dt^2} + r\left(\frac{dx_m}{dt} - \frac{dx_p}{dt}\right) + k\left(x_m - x_p\right) - F = 0 \tag{12}$$

$$Z_p \frac{dx_m}{dt} + r\left(\frac{dx_p}{dt} - \frac{dx_m}{dt}\right) + k\left(x_p - x_m\right) + F = 0 \tag{13}$$

If the force is assumed to be sinusoidal with angular frequency ω, and using the same symbols to refer to the peak values of variables, then

$F(t) \equiv Fe^{j\cdot\omega\cdot t}$ (and similarly for xm and xp)

$$-\omega^2 M_m x_m + j\omega r\left(x_m - x_p\right) + k\left(x_m - x_p\right) + F = 0 \tag{14}$$

$$j\omega Z_m x_p - j\omega r\left(x_m - x_p\right) - k\left(x_m - x_p\right) - F = 0 \tag{15}$$

or in matrix form, separating the stiffness, mass and resistance matrices

$$\left(\underline{\underline{K}} - \omega^2 \underline{\underline{M}} + j\omega \underline{\underline{R}}\right)\underline{x} - \underline{F} = 0 \quad \text{or} \quad \underline{x} = \left(\underline{\underline{K}} - \omega^2 \underline{\underline{M}} + j\omega \underline{\underline{R}}\right)^{-1} \underline{F} \tag{16}$$

where

$$\underline{\underline{M}} = \begin{pmatrix} M_m & 0 \\ 0 & 0 \end{pmatrix} \qquad \underline{\underline{K}} = \begin{pmatrix} k & -k \\ -k & k \end{pmatrix} \qquad \underline{\underline{R}} = \begin{pmatrix} r & -r \\ -r & r \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & Z_p \end{pmatrix} \tag{17a}$$

$$\underline{x} = \begin{pmatrix} x_m \\ x_p \end{pmatrix} \qquad \underline{F} = F\begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{17b}$$

So, the velocity in the panel is given by

$$Y_p = \frac{u_p}{F} = \frac{j\omega x_p}{F} \tag{18}$$

$$Y_p = j\omega \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} k - \omega^2 M_m + j\omega r & -k - j\omega r \\ -k - j\omega r & k + j\omega r + j\omega Z_p \end{pmatrix}^{-1} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{19}$$

$$Y_p = \frac{\omega^2 M_m}{\left(\omega^2 M_m \left(Z_p + r\right) - Z_p k\right) - j\omega\left(k M_m + r Z_p\right)} \tag{20}$$

By noting the velocity in the spring and damper is the difference between the velocities in the mass and panel, the equivalent circuit using the impedance analogue can be drawn as per Figure 3. It is a relatively straight forward task to verify that the ratio of panel velocity, u_p, to force, F, matches that given by the reciprocal of equation (20), i.e.

$$Zm_{eff} = Z_p\left(1 - \frac{k}{\omega^2 M_m}\right) + r - \frac{j}{\omega}\left(k + \frac{r Z_p}{M_m}\right) \tag{21}$$

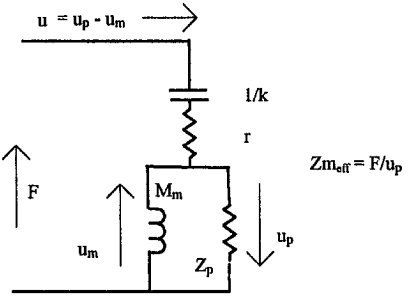


Figure 3. Impedance analogue model of DML panel.

6 Practical Implementation of the equivalent circuit for a moving coil motor

Figures 2 and 3 represent a DML panel driven by an idealised point source. If we consider the motor system to be moving coil, then M_m represents the mass of the magnet, cup and pole piece. The spring/damper represent a means of attachment of the motor to the panel. To account for the effect of the coil, we must add in series with Z_p a mechanical mass, M_c, equal to the mass of the voice coil. Additionally, the impedance Z_p is only real for a point source, and will generally be a complex quantity for a finite diameter voice coil. The reactive component, X_p, is small except at high frequencies.

Figure 4 shows a model of such a system, along with its equivalent circuit. For a complete electro-mechanical model, a gyrator and coil impedance should be added. There are computer programs on the market which facilitate such modelling, for example AkAbak [11].

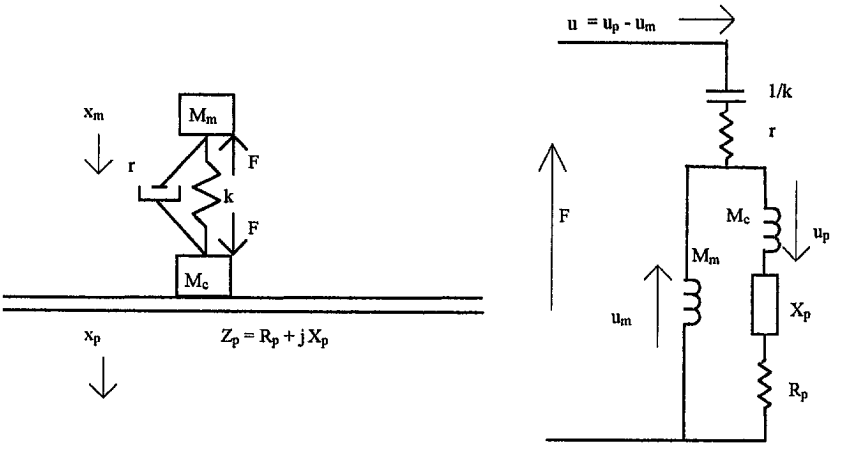


Figure 4. Impedance analogue model of DML panel with moving coil motor system.

The effective mechanical impedance relating u_p to F for Figure 4 is

$$Zm_{eff} = Z'_p\left(1-\frac{k}{\omega^2 M_m}\right)+r-\frac{j}{\omega}\left(k+\frac{rZ'_p}{M_m}\right) \tag{22}$$

where $Z'_p = R_p + jX_p + j\omega M_c$

At high frequencies, equation (22) simplifies considerably, becoming $Z'_p + r$, i.e.

$$Zm_{eff} \approx \left(R_p + r\right) + j\left(X_p + \omega M_c\right) \approx R_p + j\omega M_c \tag{23}$$

which gives the high frequency limit for the DML as

$$f_{max} \approx \frac{R_p}{2\pi M_c} \tag{24}$$

A similar simplification gives the low frequency limit which, if we ignore k, is given by

$$f_{min} \approx \frac{R_p}{2\pi M_m} \tag{25}$$

7 Conclusions

An electro-mechanical model has been presented which will enable acoustic engineers to use existing software programs to investigate the application of DML technology to their acoustic problems. Given that a stiff, light panel can be designed to have optimal modal distribution and low loss, it has been shown that in order to model the acoustic pressure or acoustic power, it is sufficient to calculate the mean velocity in the panel.

The bandwidth of the DML is seen from equations (24) and (25) to depend only on the ratio of magnet mass to coil mass. The panel properties affect the sensitivity and frequency limits only via the mechanical impedance. It is possible to design a *single* DML to be substantially flat in pressure *and* power response over a very wide bandwidth without any electrical filters. That is something which is impossible to achieve with conventional loudspeaker technology.

A more detailed model, incorporating interactions between the panel and its immediate, application specific environment is presented elsewhere at this conference [12].

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